

Mechanical department

Statistics and Numerical methods (MA42)

1. Define analysis of variance?

Analysis of variance (ANOVA) is a technique that will enable to test for the significance of difference among more than two samples mean.

2. Define experimental error?

The estimation of the amount of variation due to each of the independent factors separately and then comparing these estimates due to assignable factors with the estimate due to the chance factor is known as experimental error or simple error.

3. Write down the assumption in analysis of variance?

(i) Normality (ii) Homogeneity (iii) Independence of error?

4. Define the word treatment in analysis of variance?

The word treatment in analysis of variance is used to any factor in the experiment that is controlled at different level of values.

5. What are the three essential step to plan an experiment?

(i) A statement of the objective statement should clearly mention the hypothesis to be tested .

(ii) A description of the experiment ,and should be include the type of experimental material size of the experiment and number of replications.

(iii) The outline of the method of analysis the outline of method consists of analysis of variance.

6. Write the basic step in ANOVA?

(i) One estimate of the population variance among the sample means.

(ii) Determine a second estimate of the population variance from the variance with the sample.

(iii) Compare these estimate if they are approximately equal in value accept the null hypothesis.

7. State the formula to find SSC?

$$SSc = \sum_{j=1}^k n_j (\bar{X}_j - \bar{X})^2$$

8. Write steps to find F-rates ?

$$F = \frac{\text{Between colume variance}}{\text{Within colume variance}}$$

$$= \frac{\text{variance between samples}}{\text{variance within samples}}$$

$$= \frac{s_1^2}{s_2^2}$$

9. Write down the ANOVA table for one way classification?

10. Explain the term

Source of variation	Sum of Squares	Degree of freedom	Mean square	F-rates
Between samples	SSC	$v_1 = k - 1$	$MSc = \frac{SSc}{k-1}$	$Fc = \frac{MSc}{MSc}$
Within Samples	SSE	$v_2 = N - k$	$MSc = \frac{SSE}{N-k}$	

10. Explain the term “Homogeneity”?

The variance within each group should be equal for all group. This group is needed in order to combine or pool the variances within the groups into a single within groups source of variation.

11. Explain Independence of error /

It states that the error should be independent for each value.

12. Discuss the advantages of the two way classification method over one way classification if any ?

(i) In a two way classification method we can test two sets of hypothesis with the same data at the same but in one way classification method ,we can ‘t test two set of hypothesis.

(ii) In a one way classification method of analysis of variance the treatments constitute different levels of a single factor which is controlled in the experiment . There are however many situations in which the response variable of interest may be affected by more than one factor. That it is solved by two way classification method of a time .

13.

Explain the meaning and use of analysis of variance?

Analysis of variance to test the homogeneity several means.

Uses:

- (i) It helps to find out F-test
- (ii) Between the samples we can find the variances.

14. The total number of possibilities in which arrangements can be made in 3X3 Latin square are.....

12

15. Define design of an experiment?

The design of experiment may be defined on “ The logical construction of the experiment in which the degree of uncertainty with which the inference is drawn may be well defined .

16. Explain the purpose of design of an experiment and indicate the characteristics of a good experimental design?

Experiment can be classified into two categories

Absolute and comparative

Absolute experiments consist in determining the absolute value of some characteristics

- (i) Obtaining the average intelligence quotient of a group of people
- (ii) Finding the correlation coefficient between two variables in a bivariable distribution.

17. What are the basic principles of the design of experiments?

The principles of the experiments are

- (i) Replication
- (ii) Randomisation
- (iii) Local control

18. State the use of Chi-square distribution?

To test the significance of discrepancy between experimental values and the theoretical values .

19. What are assumptions involved in ANOVA?

- (i) Each of the samples is drawn from a normal population.
- (ii) The variance for the population from which the samples have been drawn are equal.
- (iii) The variation of each value around its own grand mean should be independent of each value

20. What are the advantages of a completely randomized experimental design ?

The use of randomized experimental design are

- (i) Easy to layout.

(ii) Allows flexibility

(iii) Simple statistical analysis

(iv) The lots of information due to missing data is smaller than with any other design.

UNIT III

1. Find an iterative formula to find \sqrt{N} where N is a positive number?

The square root of positive number N is the root of the equation

$$x^2 - N = 0$$

$$f(x) = x^2 - N$$

$$f'(x) = 2x$$

By Newton's algorithm

$$x_{k+1} = x_k - \frac{x_k^2 - N}{2x_k}$$

$$x_{k+1} = \frac{x_k^2 + N}{2x_k}$$

2. State the order of convergence and convergence condition for Newton's Raphson method ?

Soln:

$$\text{Order of convergence is } |f(x)f''(x)| < |f'(x)|^2$$

3. Why is Trapezoidal rule is so called?

Soln:

The Trapezoidal rule is so called, because it approximates the integral by the sum of n trapezoids

4. How accuracy can be increased in trapezoidal rule of evaluating the given definite integral?

If the number of points of the base segments $b - a$ is increased a better approximation to the area given by the definite integral will be obtained .

5. When does Simpson's Rule give an exact result?

Simpson's Rule will give an exact result if the entire curve $y = f(x)$ is itself a parabola.

6. State Simpson's three eighth rule?

Divide the interval into n of equal parts of size h given by $h = \frac{b-a}{n}$. Let n be a multiple of 3.

Then $\int_a^b y dx = \frac{3h}{8} [(y_1 + y_{n+1}) + 3(y_2 + y_5 + \dots + y_n) + 2(y_4 + y_7 + \dots + y_{n-2})]$

7. What is the general Newton quadratic formula? How is the trapezoidal rule its special case?

$$\int_a^{a+nh} f(x) dx = nh \left[1 + \frac{n\Delta}{2} + \frac{n(2n-3)}{12} \Delta^2 + \frac{n(n-2)^2}{24} \Delta^3 + \dots \right] f(a)$$

Put $n=1$

8. What is the order of error in Trapezoidal formula

Error in the trapezoidal formula is of the order h^2

9. What is the local error term in Simpson's one third rule

Principal part of the error in the interval $(x_1, x_2) = \frac{h^5}{90} y^{(4)}$ and $y^{(4)}$ is the value of fourth derivative of y at $x=x_1$

10. What is the local error in trapezoidal formula?

The principal part of the error in the interval $(x_1, x_2) = \frac{h^2}{12} y''$ where y'' is the value of y at x_1

11. State the Trapezoidal rule to evaluate $\int_{x_0}^{x_n} f(x) dx$?

$$\int_{x_0}^{x_n} f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$$

12. For what types of function, Simpson's rule and direct integration will give the same result?

Simpson's rule will give exact result if the entire curve is a parabola

13. What are errors in Trapezoidal and Simpson's rule of numerical integration?

Error in Trapezoidal rule

$$|E| < \frac{(b-a)^2}{12} h^2 M \text{ in the interval } (a,b) \quad h = \frac{b-a}{n}$$

Error in Simpson's rule $|E| < \frac{(b-a)^4}{180} h^4 M$

14. In order to evaluate $\int_{x_0}^{x_n} y dx$ by Simpson's one third rule as well as Simpson's three by eight rule. What is the restriction of the number of intervals?

For Simpson's one third rule the number of ordinates is odd the interval is even. For three by eight rule n is a multiple of 3.

15. What is the approximation is in deriving Simpson's rule of integration.

Simpson's one third rule approximates the area of two adjacent strips by the area under a quadratic parabola.

16. Derive Newton's algorithm for finding P^{th} root of a N?

The P^{th} root of a positive number N is the root of the equation $x^p - N = 0$

$$f(x) = x^p - N$$

$$f'(x) = px^{p-1}$$

$$X_{K+1} = x_k - \frac{x_k^p - N}{p}$$

17. Compare Gauss Jacobi and Gauss seidal method for solving linear system of the form AX=B

Gauss elimination is direct method Gauss seidle method is iterative method.

18. For solving a linear system compare Gauss elimination method and Gauss Jordan method?

In Gauss elimination method the given system is transformed with upper triangular coefficient matrix which can be solved by Back substitution method. In Gauss Jordan method the coefficient matrix reduced to a unit matrix then directly we can find the unknowns.

19. State the two differences between direct and iterative methods?

Direct method	Iterative method
It gives exact value Simple, take less time This method determines all the roots at the same time	It gives only approximate solution. Time consuming and labourious. This method determines only one root at a time.

20. Gauss seidle method is better than Gauss Jacobi?

In Gauss seidle method the latest value of unknown at each stage of iteration are used in proceeding to the next stage of iteration. Hence the convergence in Gauss seidle method is more rapid than Gauss Jacobian method.

21. What is the condition for convergence of Gauss Jacobi method of iteration?

The coefficient matrix should be diagonally dominant.

22. What are the elementary transforms?

- (i) Interchange the i^{th} and j^{th} row.
- (ii) Multiply all the elements in the i^{th} row by a number k .
- (iii) Adding the elements in the i^{th} row to the corresponding elements in the j^{th} row multiply by a constant k .

23. Explain Gauss –Elimination method to solve $= B$?

In this method the given system is transformed into an equivalent system with upper-triangular coefficient matrix.

24. Construct a linear interpolating polynomial given the points (x_0, y_0) and (x_1, y_1) ?

$$y(x) = \frac{(x-x_1)}{(x_0-x_1)}y_0 + \frac{(x-x_0)}{(x_1-x_0)}y_1$$

25. Compare Gauss-Elimination And Gauss-Seidal method?

- (i) Gauss-Elimination method advantage that it is finite and works in theory for any non singular set of equations.
- (ii) Gauss –Seidal iteration method only for special system of equations .For some system elimination is the only course available.
- (iv) In general the round of error is in iteration methods ,iteration is a self –correcting method.Any errors made at any step in the computation are corrected in the subsequent iteration.

27. Compare Gauss-Jacobi and Gauss-Seidal methods?

Gauss-Jacobi method	Gauss-Seidal method
Convergence rate is slow	The rate of convergence is roughly twice thjat of Gauss-Jacobi
Indirect method	Indirect method
Condition for convergence in the coefficient matrix is diagonally dominant	Condition for convergence in the coefficient matrix is diagonally dominant

28. Is the iterative method a self correcting method always?

In general iteration is a self correcting method since the round off error is smaller.

29. Distinguish between direct and iterative method of solving simultaneous equations?

Direct method involve a certain amount of fixed computations and they are exact solutions .Iterative or indirect methods are those in which is the solution is got by successive approximation .But the method of iteration is not applicible to all system of equations.

30. When will iterations method succeed?

In order that the iteration may succeed equation of the system must contain one large coefficient must be attached to different in that equations.This requirement will be set when the large coefficient are long the leading diagonal of the matrix of the coefficient.

31.In an iterative method the amount ofb computation depends on the degree of accuracy required state true or false?

True

32.Pickup the correct answer?

As soon as the new value for a variable is found by iteration it is used immediatly in the following equations .This method is called

(a) Gauss-seidal (b) Jacobi's (c) Gauss-Jorden (d)Relaxation.

Gauss-Seidal

33.State true or false?

The convergence in the Gauss –Seidal method in the thrice as fast as in Jacobi's method?

The statement is false .The rate of convergence of Gauss-Seidal method is roughly twice that of Gauss-Jacobi.

34.Why Gauss –Seidal method is better than Jacobi's iterative method?

Since the cvurent value of the unknowns at each stage of iteration are used in proceeding to the next stage of iteration the convergence in Gauss-Seidal method will be more repaid than in Gauss-Jacobi method.

35. Distinguish between direct and iterative method of solving simultaneous equations?

Direct	Iterative method
Can get exact solution	Only approximate solution
Simple, take less time	Time consuming laborious

36 How accuracy can be increased in a trapezoidal rule of evaluating a given definite iteration?

In the number of points of the base segment b-a is increased, a better approximation to take area given by the definite integral will be obtained.

37. Evaluate $\int_{\frac{1}{2}}^1 \frac{1}{x} dx$ by Trapezoidal rule dividing the range into 4 equal parts ?

$$\text{Here } h = \frac{1 - \frac{1}{2}}{4} = \frac{1}{8}, \quad y = \frac{1}{x}$$

x	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8}$
F(x)	$\frac{8}{4}$	$\frac{8}{5}$	$\frac{8}{6}$	$\frac{8}{7}$	$\frac{8}{8}$

$$\int_{\frac{1}{2}}^1 \frac{1}{x} dx = \frac{h}{2} [\text{sum of first and last} + 2(\text{remaining})]$$

$$= \frac{1}{16} \left[3 + \frac{856 \times 2}{210} \right] = 0.6971$$

38. Using Trapezoidal rule find $\int_0^6 f(x) dx$ from the following set of values

x	0	1	2	3	4	5	6
F(x)	1.56	3.64	4.62	5.12	7.08	9.22	10.44

$$h = 1, \quad y_1 = 1.56, \quad y_2 = 3.64$$

$A = \text{sum of the first and last term}$

$$= 1.56 + 10.44 = 12$$

$B = \text{sum of the remaining ordinate}$

$$= 3.64 + 4.62 + 5.12 + 7.08 + 9.22 = 29.68$$

$$\int_0^6 f(x) dx = \frac{1}{2} [12 + 2 \times 29.68]$$

$$= \frac{1}{2} [71.36] = 35.68$$

39. State Simpson's rule

$$\int_a^b y dx = \frac{h}{3} (A + 4B + 2C)$$

Where A=Sum of the first and last term

B= Sum of the even ordinate

C=Sum of the remaining

40. When does Simpson's rule give exact result?

Simpson's rule will give exact result if the entire curve $y = f(x)$ is itself a parabola.

41. Error in Simpson's 1/3 rule is of order ...

h^4

42. Six set of values of x and y are given (x 's being equally spaced) write the formula to get $\int_{x_1}^{x_6} y dx$?

$$\int_{x_1}^{x_6} y dx = \frac{h}{2} [(y_1 + y_6) + 2(y_2 + y_3 + y_4 + y_5)]$$

43. Six sets of values of x and y are given (x 's being equally spaced) write the formula to set $\int_{x_1}^{x_6} y dx$

$$\int_{x_1}^{x_6} y dx = \frac{h}{2} [(y_1 + y_6) + 2(y_2 + y_3 + y_4 + y_5)]$$

44. Which one more reliable Simpson's rule or Trapezoidal rule?

Simpson's rule.

45. What are the Errors in Trapezoidal and Simpson's rule of numerical integration?

Trapezoidal rule $|E| < \frac{b-a}{12} h^2 M$ h is the interval (a, b) , $h = \frac{b-a}{n}$

In Simpson's rule $|E| < \frac{b-a}{12} h^4 M$

46. In order to evaluate $\int_{x_0}^{x_n} y dx$ by Simpsons one third rule as well as by Simpsons three by eight rule what is the restriction on the number of intervals?

Let n = interval Simpsons 1/3 rule the number of ordinates is odd or even. In Simpsons 3/8 rule n should be multiple of three.

47. State true or false

Whenever Trapezoidal rule is applicable Simpson's rule can be applied

Solution: False.

48. Using Trapezoidal rule evaluate $\int_0^{\pi} \sin x dx$ by dividing the range into 6 equal parts?

Solution: Here $y(x) = \sin x$, $h = \frac{\pi}{6}$

x	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$5\frac{\pi}{6}$	π
y	0	0.5	0.866	1	0.866	0.5	0

$$\int_0^{\pi} \sin x dx = \frac{\pi}{2} [(0 + 0) + 2(0.5 + 0.866 + 1 + 0.5 + 0.866)]$$

$$= 0.622\pi$$

49. Write down the trapezoidal rule to evaluate $\int_1^6 f(x) dx$ with $h=0.5$ from the following data

X	1	1.5	2	2.5	3.0	3.5	4.0	4.5	5	5.5	6
F(x)	f(1)	f(2)	f(3)	f(4)	f(5)	f(6)	f(7)	f(8)	f(9)	f(10)	f(11)

Soln:-

$$\int_1^6 f(x) dx = \frac{h}{2} [(f(1) + f(11)) + 2(f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10))]$$

$$= \frac{0.5}{2} [(f(1) + f(11)) + 2(f(2) + f(3) + f(4) + f(5) + f(6) + f(7) + f(8) + f(9) + f(10))]$$

50. What approximation is used in deriving Simpson's rule of integration.

Solution:

Simpson's 1/3rd rule approximates the area of two adjacent strips by the area under a quadratic parabola.

51. State the basic principles of deriving Simpson's 1/3rd rule.

Solution:

The curve passing through three consecutive points is replaced by a parabola.

52. Write Milne's predictor-corrector formula?

Sol:

Milne's predictor formula is:

$$y_{n+1} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n) + 14\frac{h^5}{45}y^{(5)}(\epsilon_1) \quad \text{where } \epsilon_1 \text{ lies between } x_{n-1} \text{ and } x_{n+1}$$

$$\text{Milne's corrector formula is } y_{n+1} = y_{n-1} + \frac{h}{3}(y'_{n-1} + 4y'_n + y'_{n+1}) - \frac{h^5 y^{(5)}}{90}(\epsilon_2)$$

Where ε_2 lies between x_{n-1} and x_{n+1}

53. How many prior values are required to prior the next value in Milne's method

Four prior values.

54.The error terms in Milne's predictor formula is

$$\frac{14h}{45} \Delta^4 y_0^1$$

55. The error term in Milne's corrector formula is

$$\frac{-h}{90} \Delta^4 y_0^1$$

56.Say "True or False"

Milne's method is a self starting method

False

57.Say True or False?

Predictor—corrector methods are single step methods?

False

58.Write the convergence condition and order of convergence of Newton's-Raphson Method

$$|f(x)f^{11}(x)| < |f^1(x)|^2$$

∴The order of convergence is two.

59.Compare Simpson's one third rule with Trapezoidal method.

<i>Simpson's Rule</i>	<i>Trapezoidal Rule</i>
1.Must accurate.	1. Least accurate
2.Interval of integration must be divided into even number of subintervals.	2.can be divided into any number of sub intervals.

60. Define a diagonally dominant systems equations?

A matrix is diagonally dominant if the numerical value of the leading diagonal element in each row, is greater than or equal the sum of the numerical values of the other elements in that rows.

61.State any two properties of divided difference?

1. The divided differences are symmetrical in their arguments ie,value of any difference is independent of the order of the argument.

2.The divided difference of the sum of two function is the algebraic sum of their divided differences.

62.Gauss-Seidel method is better than he Gauss-Jacobi method why?

In Gauss-Seidel method he latest value of unknowns at each stage of iteration are used in proceeding the next stage of iteration. Hence the convergence in .Gauss-Seidel method is more rapid than Gauss-Jacobi mehod.

63.What is he condition for convergence of Gauss-Jacobi method of iteration?

The coefficient matrix should be diagonally dominant.

64. State sufficient condition for Gauss-Jacobi method to converge?

Solution: Let the equation be

$$a_1x + b_1y + c_1z = d_1, \quad a_2x + b_2y + c_2z = d_2, \quad a_3x + b_3y + c_3z = d_3$$

The sufficient condition is $|a_1| \geq |b_1| + |c_1|$

$$|b_2| \geq |a_2| + |c_2|$$

$$|c_3| \geq |a_3| + |b_3|$$

65.What are the elementary transform?

Solution: Elementary transform

(i)Interchange the i^{th} and j^{th} row or column

(ii)Multiply all the elements in the i^{th} row by a number k

(iii) Adding the elements in the i^{th} row to the corresponding elements in the j^{th} row multiply by a constant k.

66. What type of the eigen value can be obtained using power?

Solution: We can obtain dominant eigen value of the given matrix.

67. Where the Tailor's series method of solving differential equation is powerful?

Solution: If it is possible to find the successive derivatives in very easy manner then only Taylor series method is powerful.

68.Write the merits and the demerits of the Taylor method?

Solution:

Merits: 1. Taylor formula is easily derived for any order according to our interest.

2. The values of $y(x)$ or any x are easily obtained.

Demerits: This method suffers from the time consumed in calculating the higher derivatives.

69. What do we mean by saying a method is self-starting? Not self-starting?

Solution: Iteration method is self-starting since we can take a value which lies in the given interval $[a, b]$ in which the root lies. But Milne's method is not self-starting. Since we should know any 4 values prior to the value which we need.

70. Define a difference quotient?

A difference quotient is the quotient obtained by dividing the difference between two values of a function by the difference between the two corresponding values of the independent variable.

71. State the finite difference form of $\frac{\partial^2 u}{\partial y^2}$?

Solution:
$$u_{yy} = \frac{U(x_1, y+k) - 2U(x, y) + U(x, y-k)}{k^2}$$

72. In the derivation of fourth order Runge-Kutta formula why is it called fourth order?

The fourth order Runge-Kutta method agrees with Taylor series solution up to the terms of h^4

Hence it is called fourth order R-K method.

73. What are the advantages of R-K method over Taylor's method?

Solution: The Runge-Kutta method is designed to give greater accuracy and they possess the advantage of requiring only the function values at some selected points on the sub-interval.

74. Obtain the finite difference scheme for the difference equation $2 \frac{d^2 y}{dx^2} + y = 5$

Solution: $2y'' + y = 5$

Using the central difference approximation

$$2y'' = \frac{y_{k-1} - 2y_k + y_{k+1}}{h^2}$$

$$\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} + y_k = 5$$

$$y_{k-1} - 2y_k + y_{k+1} + h^2 y_k = 5h^2$$

75. Write the finite difference form of $\frac{\partial^2 u}{\partial t^2}$

Solution: $\frac{\partial^2 u}{\partial t^2} = \frac{u_{i,j-1} - 2u_{ij} + u_{i+1,j}}{k^2}$

76. State the finite differences scheme of $u_{xx} + u_{yy} = 0$

Solution: $\frac{u_{i-1,j} - 2u_{ij} + u_{i+1,j}}{h^2} + \frac{u_{i,j-1} - 2u_{ij} + u_{i,j+1}}{k^2} = 0$

When $h = k = 1$

$$u_{ij} = \frac{1}{4} [u_{i-1,j} + u_{i+1,j} + u_{i,j+1} + u_{i,j-1}]$$

1. The mean deviation is how many times the standard deviation ?

$$4/5$$

2. Define standard normal distribution ?

The normal distribution with $\alpha = 0$ $\sigma = 1$ is known as Standard normal distribution.

3. What is the lower limit of N for α^2 test ?

$$50$$

4. What is the range of F distribution?

$$1 \text{ to } \infty$$

5. What is the α^2 value of 2x2 table

a	b
c	d

$$\frac{(a+b+c+d)(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}$$

6. Define students t-test for difference of means of two samples?

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \text{ with difference } n_1 + n_2 - 2$$

$$s^2 = \frac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

7. Define standard error ?

The S D of sampling distribution of a statistic is known as standard error

8. Define null hypothesis?

For applying the test of significance we first setup of hypothesis – a definite statement about the population parameter. Such a hypothesis usually a hypothesis of no difference and if it is denoted by H_0

9. What is type I and II error?

(i) Type I error: Reject H_0 when it is true

(ii) Type II error: Accept H_0 when it is wrong

10. Define test statistics?

$$z = \frac{t - E(t)}{sE(t)}$$

11. Write the application of F-test and χ^2 test ?

F-test : To test if the samples have come from the same populations

χ^2 test: To test the significance of difference between experiment values and the theoretical values.

12. Define a F-variance?

$$F = \frac{\text{Greater variance}}{\text{smaller variance}}$$

13. Write two applications of χ^2 test?

χ^2 is used to test whether difference between observed and expected frequencies are significant.

14. Define χ^2 test of goodness of fit ?

χ^2 test of goodness of fit is a test to find if the deviation of the experiment from theory is just by chance or it is due to the of the theory to fit the observed data. By this test we test whether difference between observed and expected frequencies are significant or not.

15. What are the assumptions of t-test?

- (i) The population from which sample is drawn is normal
- (ii) The sample observations are independent, that is, sample random